3.1/10 $y = \frac{1}{x^2}$; $x_0 = 1$; $x_1 = 2$

a) avg r/c $[1,2] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

$$= \frac{f(2) - f(1)}{2-1} = \frac{\frac{1}{4} - \frac{1}{1}}{1}$$

$$= \boxed{-\frac{3}{4}}$$

average rate of change is just the slope of the segment connecting the endpoints.

b) inst r/c @ $x = x_0 = 1$

$$= \lim_{x\to 1} \frac{f(x) - f(1)}{x-1} = \lim_{x\to 1} \frac{\frac{1}{x^2} - 1}{x-1}$$

$$= \lim_{x\to 1} \frac{\frac{1}{x^2} - \frac{x^2}{x^2}}{x-1} = \lim_{x\to 1} \frac{\frac{1-x^2}{x^2}}{(x-1)}$$

$$= \lim_{x\to 1} \frac{1-x^2}{x^2(x-1)}$$

$$= \lim_{x\to 1} \frac{-(x-1)(x+1)}{x^2(x-1)}$$

$$= \lim_{x\to 1} \left(\frac{x-1}{x-1}\right)\left(\frac{-(x+1)}{x^2}\right)$$

$$= \lim_{x\to 1} \frac{-(x+1)}{x^2} = \frac{-2}{1} = -2$$

Inst. r/c is a limit of slopes

adding/subtracting fraction-like things requires a common denominator

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\cdot\left(\frac{1}{c}\right) = \frac{a}{bc}$$

$$-(a-b) = b-a$$

$$a^2 - b^2 = (a-b)(a+b)$$

group $\frac{a}{a}$ to separate 1s out of multiplication

substitution yields limit

3.1/10c $y=\frac{1}{x^2}$; $x_0=1$; $x_1=2$

c) instaneous r of c at $x=x_0$

$$=\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}=\lim_{x\to x_0}\frac{\frac{1}{x^2}-\frac{1}{x_0^2}}{x-x_0}$$

common denominator will be $x_0^2x^2$

$$=\lim_{x\to x_0}\frac{\frac{x_0^2}{x_0^2x^2}-\frac{x^2}{x_0^2x^2}}{x-x_0}=\lim_{x\to x_0}\frac{\frac{x_0^2-x^2}{x_0^2x^2}}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{x_0^2-x^2}{x_0^2x^2(x-x_0)}=\lim_{x\to x_0}\frac{-(x^2-x_0^2)}{x_0^2x^2(x-x_0)}$$

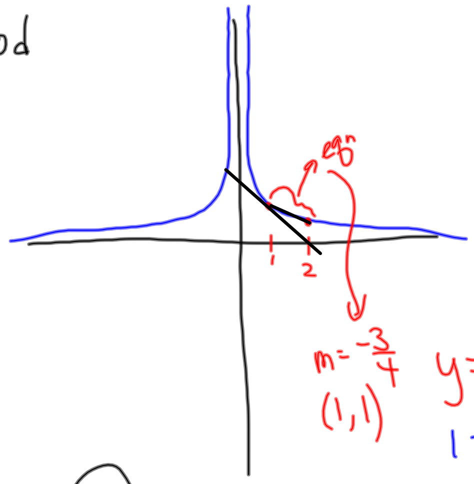
$$=\lim_{x\to x_0}\frac{-(x-x_0)(x+x_0)}{x_0^2x^2(x-x_0)}$$

$$=\lim_{x\to x_0}\frac{-(x+x_0)}{x_0^2x^2}=\frac{-(x_0+x_0)}{x_0^2(x_0^2)}$$

$\lim_{x\to x_0}$ means you are getting closer to x_0 with x, so substitute x_0 for x

$$=\frac{-2x_0}{x_0^4}=\boxed{\frac{-2}{x_0^3}}$$

3.1/10d



avg rates of chg require
* closed interval
or
* two endpoints

$m=-\frac{3}{4}$
(1,1)

$y=-\frac{3}{4}x+\frac{7}{4}$

$1=-\frac{3}{4}(1)+b$

$b=1+\frac{3}{4}=\frac{7}{4}$

on calculator

"MATH tests" have answers of TRUE $\Rightarrow$ 1

FALSE $\Rightarrow$ 0

equality
inequality signs

$$x>1\Rightarrow\begin{cases}\text{TRUE}; & \text{if } x>1\\ \text{FALSE}; & \text{if } x\le 1\end{cases}$$

31/8)

how do I factor $x^3 - a^3$?

$$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$

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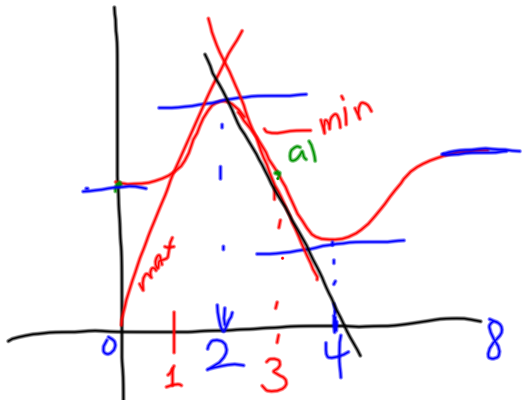
3.1/2



a) avg rate of chg betwixt 0 & 3 = 0 $\quad \frac{10-10}{3-0}=0$

b) values of t at which inst.vel. = 0

c) values of t where inst. vel. max or min
1, 3

d) -5 cm/sec
about

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3.1/ points you should take away

interval idea * average rate of chg with respect to x over interval
is just slope of secant line

point idea * instantaneous rate of change with respect to
x at $x = x_0$ is
limit of slopes between $x = x_0$ and other x's
$x \to x_0$

* geometrically, instantaneous rate of chg is
slope of tangent line.
⇒ the tangent line will be THE UNIQUE
LINE that most closely "looks like"
the original curve at a point

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Also.... DUE FRI 9/28

2.1/26,28

2.2/34,39

2.3/31,38

2.4/26

2.5/42,50

2.6/41,43,51,58

3.2/ The derivative

is "just" the FUNCTION that gives you

— instantaneous rate of change. (at any x-value)

— slope of the tangent line (at any x-value)

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the derivative

$$f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

where that limit exists

this is also the slope of the tangent line at $x = x_0$.

If the limit does not exist, the slope (and derivative are undefined.)

Alternate

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}. \quad \text{domain} = \{x \mid \text{limit exists}\}$$

Note. if $m = f'(x_0)$ then the tangent line to the curve $y = f(x)$ at $x = x_0$ is:

$$y - f(x_0) = m(x - x_0)$$